# BCS-BEC Crossover in Mix-dimensional Fermi Gases

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We investigate a mix-dimensional Fermi-Fermi mixture in which one species is confined in two-dimensional(2D) space while the other is free in three-dimensional space(3D). We determine the superfluid transition temperature $T_c$ for the entire BCS-BEC crossover including the important effects of noncondensed pairs. We find that the transition temperature reduces while the imbalance of mass is increased or lattice constant ($dz$) is reduced. In population imbalance case, the stability of superfluid is sharply destroyed by increasing the polarization.

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## 1. INTRODUCTION

Ultracold Fermi gases have attracted considerable attention [1–7] with their various tunability like the interaction strength via the Feshbach resonance and the dimensions of the system by means of optical lattices. This provides a wonderful opportunity to simulate and study the superfluidity of the fermions for entire BCS-BEC crossover. Lately, the population imbalance between the two species can also be varied. Since the imbalance causing the Fermi surface mismatch destabilizes pairing between the two species and enriches the ground state of the system [3, 4, 8–12, 14, 15]. The imbalance system possess exotic superfluid phases such as Sarma phase [14] mixture of the normal and superfluid phase, Larkin-Ovchinnikov-Fulde-Ferrel (LOFF) state [12, 13] in which the condensed pairs have nonzero net momentum.

Recently, the mix-dimensional system where the different species live in different spatial dimensions [16–19] has been realized with Bose-Bose mixture [20] by exploiting a species-selective 1D optical lattice to confine only one atomic species($^{41}K$) in 2D while the other ($^{87}Rb$) is free in the 3D space. In the mix-dimensional system, the Efimov effect [21–23] takes place only in some range of mass ratio for Fermi-Fermi mixtures [16] while for any mass ratio for Bose-Bose and Bose-Fermi mixtures at resonance. The mix-dimensional systems open a new subfield in cold atom for investigating the heteronuclear molecules whose two species constituents live in different dimensions.

In this paper we consider the two-species Fermi-Fermi mixtures in the mix-dimensional(2D-3D) system and obtain the behavior of the superfluid transition temperatures $T_c$ for the entire BCS-BEC crossover with different optical lattice parameters, the intra-species mass ratio and polarization. In addition we give the behavior of $T_c$ in resonance limit. We consider only a uniformly superfluid of the mixtures excluding from consideration the phase separated state. As for the two-species living in different spatial dimensions, the Fermi surfaces of the two-species are mismatched which also can be tuned by the parameters of the optical lattice, mass ratio and the polarizations. We analyze the superfluid of the mix-dimensional system with mismatched Fermi surfaces including the the effects of noncondensed pairs which is called "pseudogap effects" [24, 25] based on the BCS-Leggett ground state. For the nonzero temperature, the excitation gap($\Delta$) contains two contributions from pseudogap for the noncondensed pairs($\Delta_{pg}$) and the order parameter($\Delta_{sc}$). The pseudogap contribution vanishes at zero temperature while the order parameter vanish at the critical transition temperature $T_c$. The main conclusions are as follows: (a) increasing of the optical lattice parameter $dz$ (half wave length of the optical lattice) can improve the transition temperature; (b) the masses and the popularizations imbalances of the two-species both suppress the transition temperature $T_c$; (c) in population imbalance case, the superfluid phase is in a narrow region($-0.2 < p < 0.2$) and unstable for high polarization.

This paper is organized as follows. In Sec. II, the formalism of the mix-dimensional fermi gases are introduced. In Sec. III, the numerical results and discussions are given. The conclusion are given in Sec. IV.

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## 2. FORMALISM OF THE MIX-DIMENSIONAL FERMI GASES

The Hamiltonian for the mix-dimensional Fermi-Fermi mixtures system can be written as($\hbar = k_B = 1$)

$$H = \sum_{\mathbf{k},\sigma} \xi_{\mathbf{k},\sigma} a^\dagger_{\mathbf{k},\sigma} a_{\mathbf{k},\sigma} + g \sum_{\mathbf{k},\mathbf{k'},\mathbf{q}} a^\dagger_{\mathbf{q/2}+\mathbf{k},\uparrow} a^\dagger_{\mathbf{q/2}-\mathbf{k},\downarrow} a_{\mathbf{q/2}-\mathbf{k'},\downarrow} a_{\mathbf{q/2}+\mathbf{k'},\uparrow}, \tag{1}$$

where, the pseudo-spin $\sigma = \uparrow, \downarrow$ labels the two types of Fermi atoms. We assume that the $\uparrow$ Fermi atoms is confined in a species-selective 1D optical lattice while the other is free in the 3D. The free dispersions of the two species Fermi atoms are $\xi_{\mathbf{k},\uparrow} = \frac{k_x^2+k_y^2}{2m_\uparrow} + 2t[1 - cos(k_z d_z)] - \mu_\uparrow$ and $\xi_{\mathbf{k},\downarrow} = \frac{k_x^2+k_y^2+k_z^2}{2m_\downarrow} - \mu_\downarrow$. Here $\mu_\sigma$ is the two species Fermi atoms chemical potential, $t$ is the amplitude of the $\uparrow$ Fermi atoms tunnelling to the nearest-neighbor sites, and $d_z$ is the lattice parameter which is half wave length of the 1D optical lattice.

We determine the transition temperature $T_c$ including the pair fluctuation (pseudogap effect). Truncating the equation of motion for Green's functions scheme we can get the pair propagator

$$t(Q) = \frac{g}{1 + g\chi(Q)}, \tag{2}$$

and the self-energy

$$\Sigma_\sigma(K) = \sum_Q t(Q) G_{0,\sigma}(Q - K), \tag{3}$$

where the pair susceptibility is given by

$$\chi(Q) = \frac{1}{2}[\chi_{\uparrow\downarrow}(Q) + \chi_{\downarrow\uparrow}(Q)] = \frac{1}{2}\sum_K [G_{0,\uparrow}(Q - K)G_\downarrow(K) + G_{0,\downarrow}(Q - K)G_\uparrow(K)], \tag{4}$$

with the bare green's function $G_{0,\sigma}^{-1}(K) = \imath\omega_n - \xi_{\mathbf{k},\sigma}$, and the $G_\sigma(K)$ is dressed green's function. We take the notation $K = (\imath\omega_n, \mathbf{k})$, $Q = (\imath\Omega_n, \mathbf{q})$, $\Sigma_K = T\Sigma_n\Sigma_\mathbf{k}$, etc., where $\omega_n(\Omega_n)$ is the odd(even) Matsubara frequency.

Below the critical temperature $T_c$, the $\mathbf{T}$ matrix and self energy contains both the condensed($sc$) and the noncondensed or "pseudogap"-associated($pg$) contributions:

$$t(Q) = t_{sc}(Q) + t_{pg}(Q), \tag{5}$$

$$t_{sc}(Q) = -\frac{\Delta_{sc}}{T}\delta(Q), \tag{6}$$

$$t_{pg}(Q) = \frac{g}{1 + g\chi(Q)}, \qquad Q \neq 0. \tag{7}$$

So the total fermion self-energy is given by

$$\Sigma_\sigma(K) = \Sigma_{\sigma,sc}(K) + \Sigma_{\sigma,pg}(K) = -\Delta^2 G_{0,\bar\sigma}(-K), \tag{8}$$

where the excitation gap contain two contributions of the order parameter ($sc$) and the pseudogap($pg$) $\Delta^2 = \Delta_{sc}^2 + \Delta_{pg}^2$, with the pseudogap $\Delta_{pg}^2 = -\sum_{Q\neq0} t_{pg}(Q)$.

The dressed green's function $G_\sigma$ can be derived by Dyson equation

$$G_\uparrow(K) = \frac{u_\mathbf{k}^2}{\imath\omega_n - \xi_\mathbf{k}^\alpha} + \frac{v_\mathbf{k}^2}{\imath\omega_n + \xi_\mathbf{k}^\beta}, \tag{9}$$

$$G_\downarrow(K) = \frac{u_\mathbf{k}^2}{\imath\omega_n - \xi_\mathbf{k}^\beta} + \frac{v_\mathbf{k}^2}{\imath\omega_n + \xi_\mathbf{k}^\alpha}, \tag{10}$$

where, the fermion excitation $\xi_\mathbf{k}^{\alpha,\beta} = E_\mathbf{k} \pm \xi_\mathbf{k}^-$, with $E_\mathbf{k} = \sqrt{\xi_\mathbf{k}^{+2} + \Delta^2}$, $\xi_\mathbf{k}^\pm = (\xi_{\mathbf{k},\uparrow} \pm \xi_{\mathbf{k},\downarrow})/2$, $u_\mathbf{k}^2 = (1 + \xi_\mathbf{k}^+/E_\mathbf{k})/2$, $v_\mathbf{k}^2 = (1 - \xi_\mathbf{k}^+/E_\mathbf{k})/2$.

In superfluid state, the "gap equation" is given by the Thouless criterion, $t^{-1}(Q = 0) = 0$, which is equivalent to the BEC condition of pairs, $\mu_{pair} = 0$. The coupling constant $g$ is written in terms of the s-wave scattering length $a$, via the relationship $M/(4\pi a) = 1/g + \Sigma_{\mathbf{k}}(2\epsilon_{\mathbf{k}}^+)$, where $M = 2m_\uparrow m_\downarrow/(m_\uparrow + m_\downarrow)$ and $\epsilon^\pm = (\epsilon_\uparrow \pm \epsilon_\uparrow)/2$. The gap equation reduce to

$$-\frac{M}{2\pi a} = \sum_{\mathbf{k}}[\frac{1 - f(\xi_{\mathbf{k}}^\alpha) - f(\xi_{\mathbf{k}}^\beta)}{2E_{\mathbf{k}}} - \frac{1}{\epsilon_{\mathbf{k}}^+}], \tag{11}$$

where the $f(x)$ is the Fermi distribution function. The number of $\sigma$ fermion: $n_\sigma = \Sigma_K G_\sigma(K)$. The equations of the total number $n = n_\uparrow + n_\downarrow$ and the number difference $pn = n_\uparrow - n_\downarrow$ of fermions are

$$n = \sum_{\mathbf{k}}[2v_{\mathbf{k}}^2 + \frac{\xi_{\mathbf{k}}^+}{E_{\mathbf{k}}}(f(\xi_{\mathbf{k}}^\alpha) + f(\xi_{\mathbf{k}}^\beta))], \tag{12}$$

$$pn = \sum_{\mathbf{k}}[f(\xi_{\mathbf{k}}^\alpha) - f(\xi_{\mathbf{k}}^\beta)], \tag{13}$$

here, $p$ is polarization of the two-species Fermi atoms. Since the contribution to pseudogap is dominated by the small $Q$ divergent region for the BEC condition. The $T$ matrix can be expanded in low energy and long wavelength limit. The equation of pseudogap can be written as

$$\Delta_{pg}^2 = -\sum_Q t_{pg}(Q) = Z^{-1}\sum_{\mathbf{q}} b(\Omega_{\mathbf{q}}), \tag{14}$$

where, $\Omega_{\mathbf{q}} = \frac{\mathbf{q}_\perp^2}{2M_\perp^*} + \frac{\mathbf{q}_z^2}{2M_z^*}$ is pair dispersion with $M_\perp^*$ and $M_z^*$ representing the anisotropic effective pair mass computed from the low energy and long wavelength expansion of the pair susceptibility $\chi(Q)$[given in Appendix] and $b(\Omega_{\mathbf{q}})$ is the boson distribution.

The stability of the superfluid phase requires the positive definiteness of the number susceptibility matrix, which is equivalent to the positive second-order partial derivation of the thermodynamical potential with respect to the excitation gap $(\partial^2\Omega/\partial\Delta^2)_{\mu_\uparrow,\mu_\downarrow} > 0$

$$(\frac{\partial^2\Omega}{\partial\Delta^2})_{\mu_\uparrow,\mu_\downarrow} = 2\sum_{\mathbf{k}}\frac{\Delta^2}{E_{\mathbf{k}}^2}[\frac{1 - f(\xi_{\mathbf{k}}^\alpha) - f(\xi_{\mathbf{k}}^\beta)}{2E_{\mathbf{k}}} + \frac{f'(\xi_{\mathbf{k}}^\alpha) + f'(\xi_{\mathbf{k}}^\beta)}{2}] > 0, \tag{15}$$

In addition, we also consider the positivity of the anisotropic effective pair mass $M_z^*$ and $M_\perp^*$. Eqs.(11)-(14) are closed for our mix-dimensional system. For $T = 0$, the pseudogap contribution vanish so the excitation gap ($\Delta = \Delta_{sc}$). For $T > T_c$, the order parameter ($\Delta_{sc}$) vanish such that the excitation gap contains the pseudogap only ($\Delta = \Delta_{pg}$). The transition temperature $T_c$ can be determined by self-consistently solving this closed equations throughout the entire BCS-BEC crossover with $\Delta_{sc}^2 = 0$.

## 3. NUMERICAL RESULTS AND DISCUSSIONS

First, the transition temperature $T_c$, are obtained from the self-consistent equations, as a function of scattering length, $1/k_F a$, for different optical lattice parameter, $k_F dz$, with the tunnelling amplitude $t = E_F$ in population symmetry case ($p = 0$), shown in Fig.[1]. In the BCS limit, $1/k_F a \to -\infty$, the transition temperature $T_c$ as well as the anisotropic effective mass $M_z$ and $M_\perp$ of the cooper pairs is reduce to zero. As the interaction increase to the unitarity, $T_c$ rises rapidly. In the BEC limit, $1/k_F a \to \infty$, $T_c$ varies very slowly and the anisotropic effective mass of the pairs $M_\perp^* = 2m$ while $M_z^*$ depends on the value of $k_F dz$. Increasing the parameter $dz$, lattice constant of the 1D optical lattice, improves the transition temperature for the entire BCS-BEC crossover. This phenomenon can be illustrated as follows. When the tunnelling amplitude $t = E_F$, the Fermi surface of the $\downarrow$ is a spherical shell while the $\uparrow$ is a prolate spheroid. Increasing of the parameter $k_F dz$ can decrease the mismatch of the two Fermi surface such that increasing $k_F dz$ is propitious to form the cooper pairs. If the parameter $dz$ is much larger than the inverse of Fermi momentum $k_F^{-1}$, the localization of the $\uparrow$ fermion in $\hat{\mathbf{z}}$ direction is very weak and the Mix-dimensional effect is reduced. Hence the parameter $dz$ must not be much larger than the inverse of Fermi momentum $k_F^{-1}$.
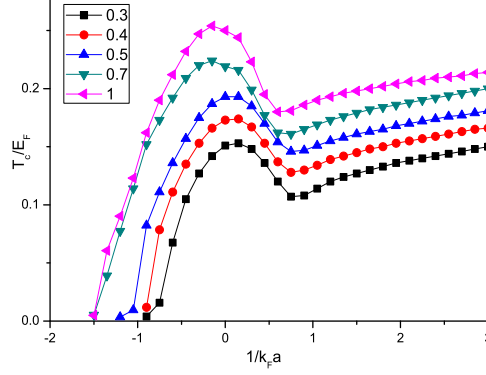
FIG. 1: (color online). $T_c$ as a function of $1/k_F a$ for different lattice parameter $k_F dz$ with the polarization $p = 0$; $t = E_F = \hbar^2 k_F^2/2m$ and $m_\uparrow = m_\downarrow$, Here $k_F$ is the noninteracting Fermi momentum for polarization $p = 0$.
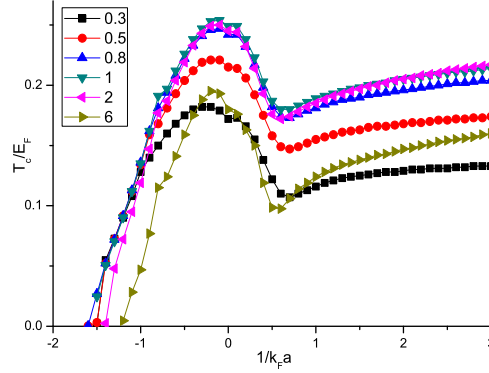


FIG. 2: (color online). Self-consistent solutions of the transition temperature $T_c$ as a function of $1/k_F a$ for different mass ratio $\eta = m_\uparrow/m_\downarrow$ with the polarization $p = 0$; $t = E_F = \hbar^2 k_F^2/2m$ and $k_F dz = 1$. The transition temperature $T_c$ reduce as the mass imbalance increasing.

Second, the transition temperature $T_c$, are obtained from the self-consistent equations, as a function of $1/k_F a$ for different mass ratio $\eta$, shown in fig.[2]. We consider the population symmetry, the tunnelling amplitude $t = E_F$ and the parameter $k_F dz = 1$. In terms of the mixed dimensions system of Fermi-Fermi mixture(2D-3D), the system is stable against the Efimov effect[16, 22, 23] for even-parity case.

Then, the transition temperature $T_c$ attains the maximum at the mass balance case and reduces for the mass ratio deviating from the symmetry case as shown in the Fig.[3] at resonance limit. The imbalance of the mass disturbs the inter-species pairing.

Finally, for the population imbalance case, we find that the superfluid phase is restricted in the region of low polarization $-0.2 < p < 0.2$ at resonance, shown in Fig.[4]. The transition temperature $T_c$ reduces while the polarization is increased. The superfluid phase in the BEC limit is not stable for the condition of stability is violated as $(\partial^2 \Omega/\partial \Delta^2)_{\mu_\uparrow, \mu_\downarrow} < 0$. At high polarization, the superfluid is unstable in the entire BCS-BEC region which is very different from the pure three dimensions systems [26]. The superfluid of the Fermi-Fermi mixture system in mixed dimensions is very unstable with population imbalance. In BCS side the anisotropic effective mass smoothly reduce to zero. In the unstable regions, other possible phase like the LOFF state maybe occur which will be consider in the future works.
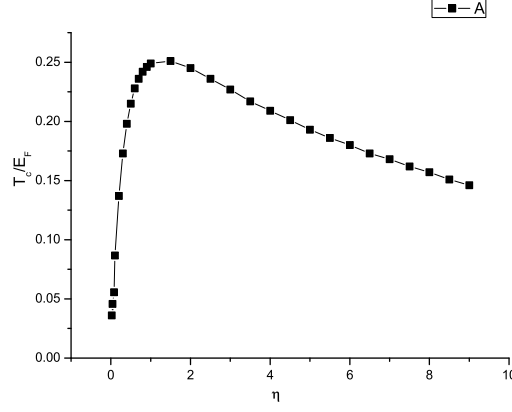
FIG. 3: $T_c$ as a function of the mass ratio $\eta = m_\uparrow/m_\downarrow$ at resonance $1/k_F a = 0$ with the polarization $p = 0$; $t = E_F = \hbar^2 k_F^2/2M$ and $k_F dz = 1$.
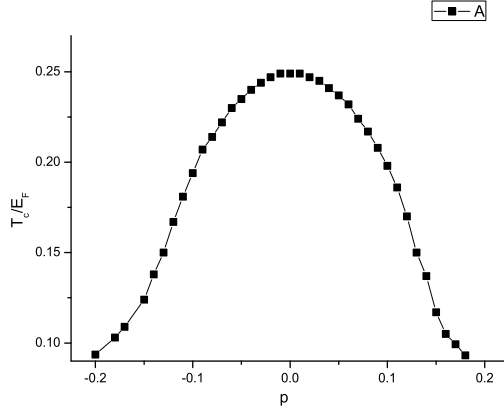


FIG. 4: $T_c$ as a function of the polarization $p = (n_\uparrow - n_\downarrow)/n$ at resonance $1/k_F a = 0$ in the case of $t = E_F = \hbar^2 k_F^2/2m$; $k_F dz = 1$ and $m_\uparrow = m_\downarrow$.

## 4.  CONCLUSION

To summarize, we obtain the transition temperature of the Fermi-Fermi mixture in Mix-dimensional system, as a function of the interaction strength for different optical lattice parameter $dz$ and mass ratio in the BCS-BEC crossover. In addition, we also obtain the transition temperature behavior in resonance for different mass ratio and polarization. We obtain the transition temperature include the effect of the noncondensed cooper pairs by the generalized mean field theory based on the BCS-Leggett ground state in $G_0 G$ schema. Increasing of the optical lattice parameter $dz$ have positive effects to the transition temperature. In BEC limit, the anisotropic effective mass $M_\perp = 2m$ for different $dz$. In mass case, the transition temperature has maximum value at mass symmetry and reduce while the masses deviate from the balance. Superfluid phase is restricted within a narrow region of low polarization in population imbalance case.

### Acknowledgments

## Appendix:

In this appendix, we evaluate the expansion of the pair susceptibility in low energy and long wavelength limit and derive the explicit expression of the anisotropic effective mass of the cooper pairs.

The pair susceptibility is given by

$$\chi(Q) = \frac{1}{2}[\chi_{\uparrow\downarrow}(Q) + \chi_{\downarrow\uparrow}(Q)] = \frac{1}{2}\sum_K [G_{0,\uparrow}(Q-K)G_\downarrow(K) + G_{0,\downarrow}(Q-K)G_\uparrow(K)], \tag{16}$$

The explicit expression of the pair susceptibility can be got by substitute the Green's functions

$$
\begin{aligned}
\chi(Q) &= \frac{1}{2}\sum_{\mathbf{k}}\Big[\frac{1 - f(\xi_{\mathbf{k}}^\beta) - f(\xi_{\mathbf{k}-\mathbf{q},\uparrow})}{\xi_{\mathbf{k}}^\beta + \xi_{\mathbf{k}-\mathbf{q},\uparrow} - \imath\Omega_n}u_{\mathbf{k}}^2 - \frac{f(\xi_{\mathbf{k}}^\alpha) - f(\xi_{\mathbf{k}-\mathbf{q},\uparrow})}{\xi_{\mathbf{k}}^\alpha - \xi_{\mathbf{k}-\mathbf{q},\uparrow} + \imath\Omega_n}v_{\mathbf{k}}^2 \\
&\quad + \frac{1 - f(\xi_{\mathbf{k}}^\alpha) - f(\xi_{\mathbf{k}-\mathbf{q},\downarrow})}{\xi_{\mathbf{k}}^\alpha + \xi_{\mathbf{k}-\mathbf{q},\downarrow} - \imath\Omega_n}u_{\mathbf{k}}^2 - \frac{f(\xi_{\mathbf{k}}^\beta) - f(\xi_{\mathbf{k}-\mathbf{q},\downarrow})}{\xi_{\mathbf{k}}^\alpha - \xi_{\mathbf{k}-\mathbf{q},\downarrow} + \imath\Omega_n}v_{\mathbf{k}}^2\Big],
\end{aligned}
\tag{17}
$$

At transition temperature $T_c$, $t_{sc}^{-1}(Q) = 0$ and

$$t_{pg}^{-1}(Q) = t^{-1}(Q) = \chi(Q) - \chi(0), \tag{18}$$

In low energy and long wavelength limit

$$t_{pg}^{-1}(Q) = Z(\imath\Omega_n - \Omega_{\mathbf{q}} + \mu_{pair} + \imath\Gamma_{q,\Omega}), \tag{19}$$

$\Omega_{\mathbf{q}} = \frac{q_\perp^2}{2M_\perp^*} + \frac{q_z^2}{2M_z^*}$ is the dispersion of the cooper pairs. $M_z$ and $M_\perp$ is the anisotropic effective mass. Below the critical temperature $T_c$ the chemical potential of cooper pair vanish $\mu_{pair} = 0$ as for the BEC condition. The imaginary part of the pair dispersion is given by

$$
\begin{aligned}
\Gamma_{q,\Omega} &= \frac{\pi}{Z}\sum_{\mathbf{k}}\{[1 - f(\xi_{\mathbf{k}}^\beta) - f(\xi_{\mathbf{k}-\mathbf{q},\uparrow})]u_{\mathbf{k}}^2\delta(\xi_{\mathbf{k}}^\beta + \xi_{\mathbf{k}-\mathbf{q},\uparrow} - \Omega) + [f(\xi_{\mathbf{k}}^\alpha) - f(\xi_{\mathbf{k}-\mathbf{q},\uparrow})]v_{\mathbf{k}}^2\delta(\xi_{\mathbf{k}}^\alpha - \xi_{\mathbf{k}-\mathbf{q},\uparrow} + \Omega) \\
&\quad + [1 - f(\xi_{\mathbf{k}}^\alpha) - f(\xi_{\mathbf{k}-\mathbf{q},\downarrow})]u_{\mathbf{k}}^2\delta(\xi_{\mathbf{k}-\mathbf{q},\downarrow} - \Omega) + [f(\xi_{\mathbf{k}}^\beta) - f(\xi_{\mathbf{k}-\mathbf{q},\downarrow})]v_{\mathbf{k}}^2\delta(\xi_{\mathbf{k}}^\alpha - \xi_{\mathbf{k}-\mathbf{q},\downarrow} + \Omega)\},
\end{aligned}
\tag{20}
$$

and the reverse of the residue is

$$
\begin{aligned}
Z &= \frac{\partial\chi}{\partial\imath\Omega}\Big|_{\Omega=\mathbf{q}=0} \\
&= \frac{1}{2\Delta^2}[n - \Sigma_{\mathbf{k}}(f(\xi_{\mathbf{k},\uparrow}) + f(\xi_{\mathbf{k},\downarrow}))],
\end{aligned}
\tag{21}
$$

The anisotropic effective masses of the cooper pair are given by

$$
\begin{aligned}
\frac{1}{2M_\perp^*} &= -\frac{1}{4Z}\frac{\partial^2 t_{pg}^{-1}}{\partial\mathbf{q}_\perp^2}\Big|_{\Omega=\mathbf{q}=0} \\
&= -\frac{1}{8Z}\sum_{\mathbf{k}}\Big\{\frac{2f'(\xi_{\mathbf{k},\uparrow})}{\Delta^2}\Big(\frac{\partial\xi_{\mathbf{k},\uparrow}}{\partial\mathbf{k}_\perp}\Big)^2 + \frac{2f'(\xi_{\mathbf{k},\downarrow})}{\Delta^2}\Big(\frac{\partial\xi_{\mathbf{k},\downarrow}}{\partial\mathbf{k}_\perp}\Big)^2 \\
&\quad - \frac{(1 - f(\xi_{\mathbf{k}}^\beta) - f(\xi_{\mathbf{k}}^\uparrow))(E_{\mathbf{k}} - \xi_{\mathbf{k}}^+) + (f(\xi_{\mathbf{k}}^\alpha) - f(\xi_{\mathbf{k}}^\uparrow))(E_{\mathbf{k}} + \xi_{\mathbf{k}}^+)}{2E_{\mathbf{k}}\Delta^2}\Big(\frac{\partial^2\xi_{\mathbf{k},\uparrow}}{\partial\mathbf{k}_\perp^2}\Big) \\
&\quad - \frac{(1 - f(\xi_{\mathbf{k}}^\alpha) - f(\xi_{\mathbf{k}}^\downarrow))(E_{\mathbf{k}} - \xi_{\mathbf{k}}^+) + (f(\xi_{\mathbf{k}}^\beta) - f(\xi_{\mathbf{k}}^\downarrow))(E_{\mathbf{k}} + \xi_{\mathbf{k}}^+)}{2E_{\mathbf{k}}\Delta^2}\Big(\frac{\partial^2\xi_{\mathbf{k},\downarrow}}{\partial\mathbf{k}_\perp^2}\Big) \\
&\quad + \frac{(1 - f(\xi_{\mathbf{k}}^\beta) - f(\xi_{\mathbf{k}}^\uparrow))(E_{\mathbf{k}} - \xi_{\mathbf{k}}^+)^2 - (f(\xi_{\mathbf{k}}^\alpha) - f(\xi_{\mathbf{k}}^\uparrow))(E_{\mathbf{k}} + \xi_{\mathbf{k}}^+)^2}{E_{\mathbf{k}}\Delta^4}\Big(\frac{\partial\xi_{\mathbf{k},\uparrow}}{\partial\mathbf{k}_\perp}\Big)^2 \\
&\quad + \frac{(1 - f(\xi_{\mathbf{k}}^\alpha) - f(\xi_{\mathbf{k}}^\downarrow))(E_{\mathbf{k}} - \xi_{\mathbf{k}}^+)^2 - (f(\xi_{\mathbf{k}}^\beta) - f(\xi_{\mathbf{k}}^\downarrow))(E_{\mathbf{k}} + \xi_{\mathbf{k}}^+)^2}{E_{\mathbf{k}}\Delta^4}\Big(\frac{\partial\xi_{\mathbf{k},\downarrow}}{\partial\mathbf{k}_\perp}\Big)^2\Big\},
\end{aligned}
$$

$$\frac{1}{2M_z^*} = -\frac{1}{2Z}\frac{\partial^2 t_{pg}^{-1}}{\partial k_z^2}\mid_{\Omega=\mathbf{q}=0}$$

$$= -\frac{1}{4Z}\sum_{\mathbf{k}}\{\frac{2f'(\xi_{\mathbf{k},\uparrow})}{\Delta^2}(\frac{\partial\xi_{\mathbf{k},\uparrow}}{\partial k_z})^2 + \frac{2f'(\xi_{\mathbf{k},\downarrow})}{\Delta^2}(\frac{\partial\xi_{\mathbf{k},\downarrow}}{\partial k_z})^2$$

$$-\frac{(1-f(\xi_{\mathbf{k}}^{\beta})-f(\xi_{\mathbf{k}}^{\uparrow}))(E_{\mathbf{k}}-\xi_{\mathbf{k}}^{+})+(f(\xi_{\mathbf{k}}^{\alpha})-f(\xi_{\mathbf{k}}^{\uparrow}))(E_{\mathbf{k}}+\xi_{\mathbf{k}}^{+})}{2E_{\mathbf{k}}\Delta^2}\frac{\partial^2\xi_{\mathbf{k},\uparrow}}{\partial k_z^2}$$

$$-\frac{(1-f(\xi_{\mathbf{k}}^{\alpha})-f(\xi_{\mathbf{k}}^{\downarrow}))(E_{\mathbf{k}}-\xi_{\mathbf{k}}^{+})+(f(\xi_{\mathbf{k}}^{\beta})-f(\xi_{\mathbf{k}}^{\downarrow}))(E_{\mathbf{k}}+\xi_{\mathbf{k}}^{+})}{2E_{\mathbf{k}}\Delta^2}\frac{\partial^2\xi_{\mathbf{k},\downarrow}}{\partial k_z^2}$$

$$+\frac{(1-f(\xi_{\mathbf{k}}^{\beta})-f(\xi_{\mathbf{k}}^{\uparrow}))(E_{\mathbf{k}}-\xi_{\mathbf{k}}^{+})^2-(f(\xi_{\mathbf{k}}^{\alpha})-f(\xi_{\mathbf{k}}^{\uparrow}))(E_{\mathbf{k}}+\xi_{\mathbf{k}}^{+})^2}{E_{\mathbf{k}}\Delta^4}(\frac{\partial\xi_{\mathbf{k},\uparrow}}{\partial k_z})^2$$

$$+\frac{(1-f(\xi_{\mathbf{k}}^{\alpha})-f(\xi_{\mathbf{k}}^{\downarrow}))(E_{\mathbf{k}}-\xi_{\mathbf{k}}^{+})^2-(f(\xi_{\mathbf{k}}^{\beta})-f(\xi_{\mathbf{k}}^{\downarrow}))(E_{\mathbf{k}}+\xi_{\mathbf{k}}^{+})^2}{E_{\mathbf{k}}\Delta^4}(\frac{\partial\xi_{\mathbf{k},\downarrow}}{\partial k_z})^2\},$$

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